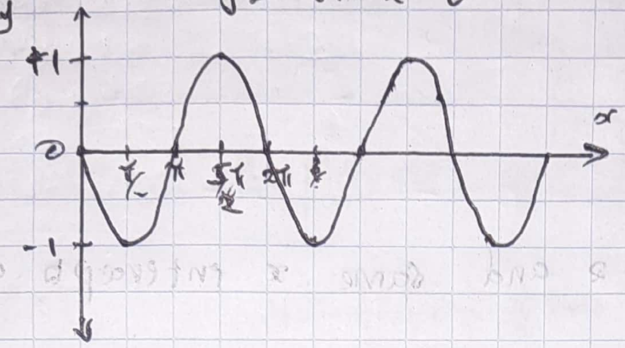
Section 4.1 (*) Note on the measurement of $y(1, x)$

Match the sinusoidal fxn with its graph.

a) $y = -\sin x$.

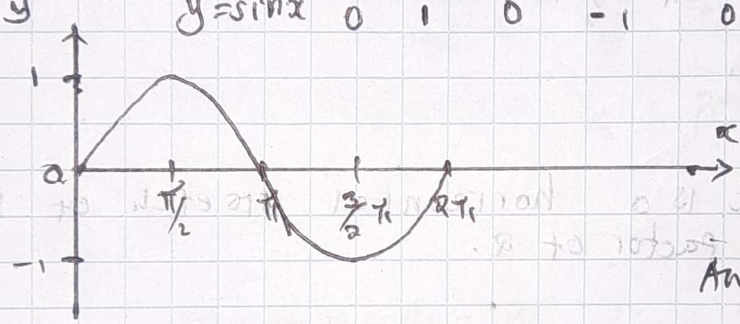
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	1	0	-1	0
$y = -\sin x$	0	-1	0	1	0



Answer: c.

b) $y = \sin x$.

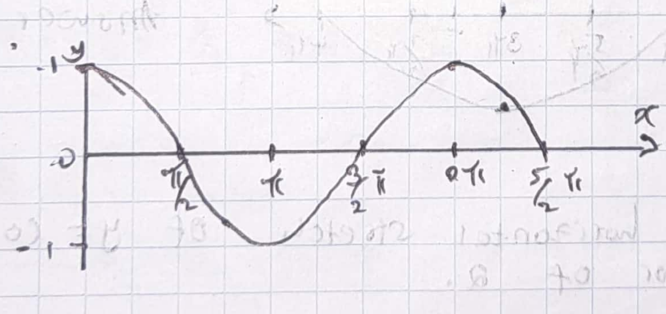
x	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π	$\frac{5}{2}\pi$
$y = \sin x$	0	1	0	-1	0	1



Answer: d

3) $y = \cos x$.

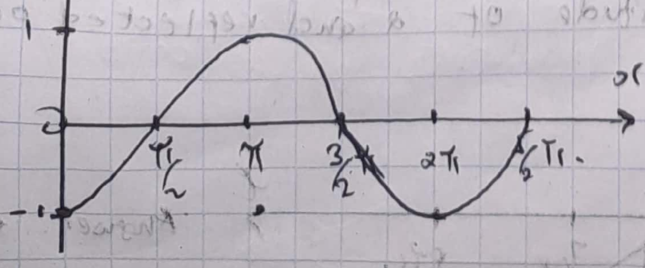
x	$y = \cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3}{2}\pi$	0
2π	1



Answer (a)

4 $y = -\cos x$.

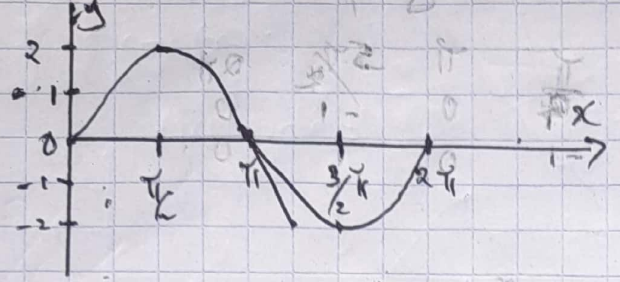
It is the graph of $y = \cos x$ reflected on x-axis.



Answer: j

5) $y = 2\sin x$.

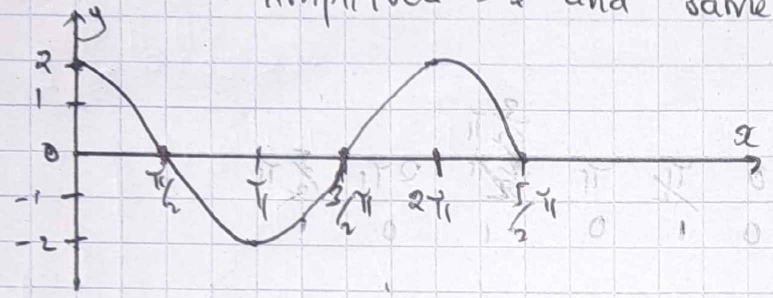
Amplitude = 2. and has same intercepts as $y = \sin x$



Answer: h.

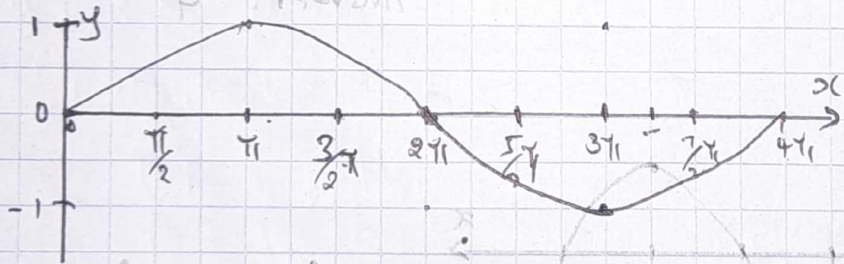
6) $y = 2\cos x$.

Amplitude = 2 and same x intercepts as $y = \cos x$.



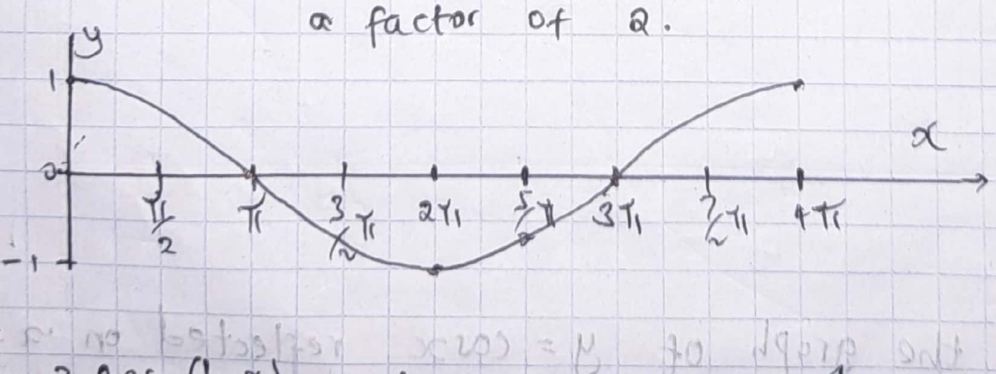
Answer: i

7) $y = \sin(\frac{1}{2}x)$ It is a horizontal stretch of $y = \sin x$ by a factor of 2.



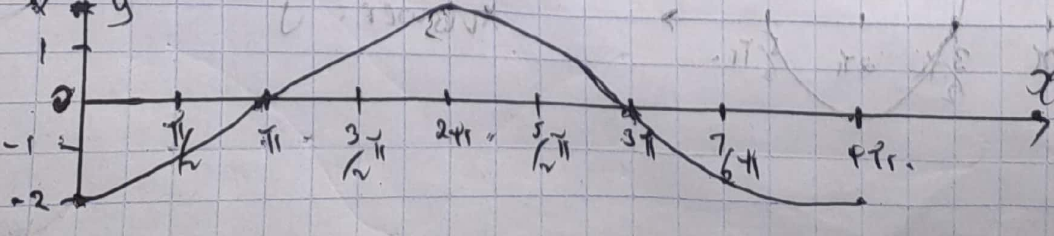
Answer: b.

8. $y = \cos(\frac{1}{2}x)$ It is a horizontal stretch of $y = \cos x$ by a factor of 2.



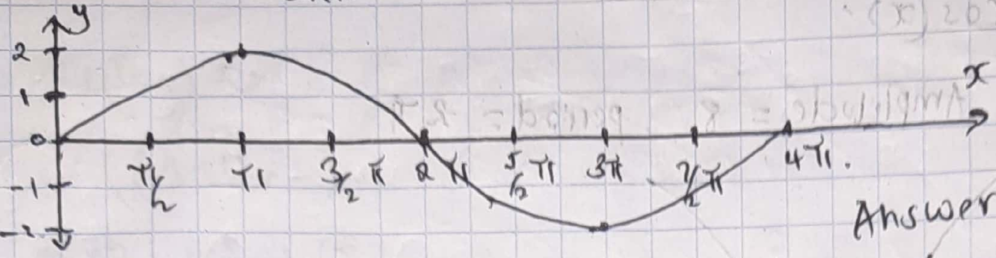
Answer: f.

9. $y = -2\cos(\frac{1}{2}x)$ It has the same shape as $y = \cos(\frac{1}{2}x)$ but has an amplitude of 2 and reflected on x axis.



Answer: e.

10) $y = -2\sin(\tfrac{1}{2}x)$ It has the same shape as $y = \sin(\tfrac{1}{2}x)$ but has an amplitude of 2 and reflected on x axis.



Answer: g.

(11-24) => state Amplitude and period.

11) $y = \frac{3}{2}\cos(3x)$ ~~$y = |A|\cos(Bx)$~~ $y = A\cos(Bx)$.

~~Periode = A~~ Amplitude $= A = \frac{3}{2}$.

period $= \frac{2\pi}{|B|} = \frac{2\pi}{3} = 2\frac{\pi}{3}$.

Answer: Amplitude $= \frac{3}{2}$, period $= \frac{2}{3}\pi$

15) $y = \frac{2}{3}\cos(\frac{3}{2}x)$

$|A| =$ amplitude $= \frac{2}{3}$.

period: ~~period~~: $B = \frac{3}{2}$ and $\frac{2\pi}{|B|} = \frac{2\pi}{\frac{3}{2}} = \frac{4}{3}\pi$

19) $y = 5\sin(\frac{\pi}{3}x)$

Amplitude $= |A| = 5$

period $= \frac{2\pi}{|B|} = \frac{2\pi}{\pi/3} = 6\pi$

23) $y = -\frac{1}{3}\sin(\frac{1}{4}x)$

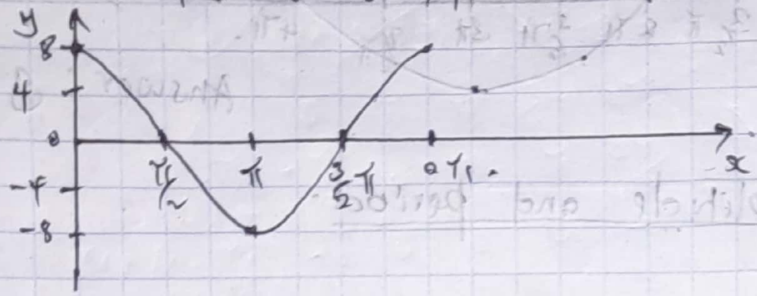
Amplitude $= |A| = |-\frac{1}{3}| = \frac{1}{3}$

period $= \frac{2\pi}{|B|} = \frac{2\pi}{\frac{1}{4}} = 8\pi$

(25 - 40) : Graph the fxns over 1 period.

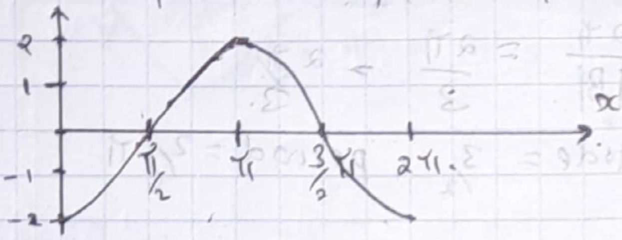
25) $y = 8\cos(x)$.

Amplitude $= 8$, period $= 2\pi$



29) $y = -2\cos(x)$.

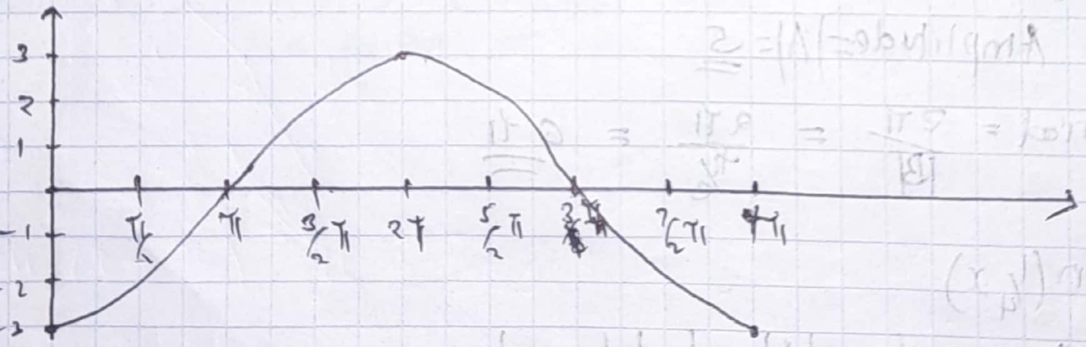
Amplitude $= |-2| = 2$, period $= 2\pi$: Reflection of $y = \cos x$ over x axis.



33) $y = -3\cos(\frac{1}{2}x)$

Amplitude $= |-3| = 3$ period $= \frac{2\pi}{1/2} = 4\pi$.

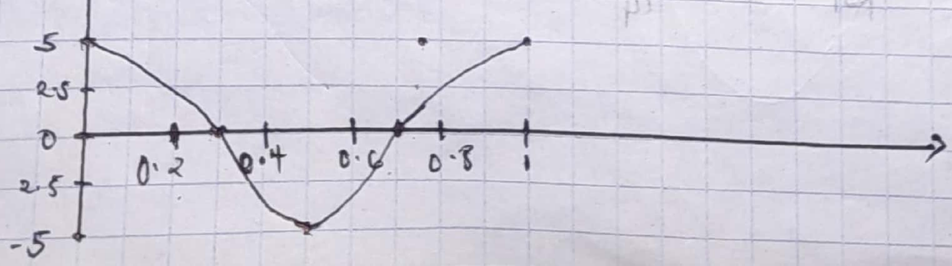
It is a horizontal stretch of $y = \cos x$ on a scale factor of 2, vertical stretch scale factor of 3 and Reflected along the x-axis.



37) $y = 5\cos(2\pi x)$

Amplitude $= 5$ period $= \frac{2\pi}{2\pi} = 1$

x intercepts: $\left(\frac{2n+1}{4}\right) = (1/4, 0)$ and $(3/4, 0) \Rightarrow (0.25, 0)\ (0.75, 0)$



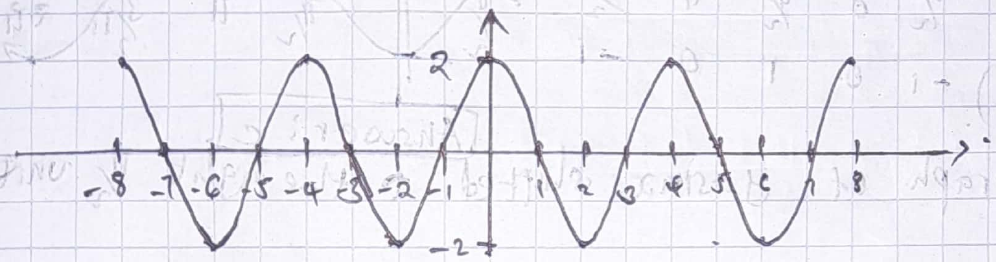
41) $y = 2\cos(\frac{\pi}{2}x)$

Amplitude = 2 period = $\frac{2\pi}{\pi/2} = 4$.

When $\cos(\frac{\pi}{2}x) = 0$ ∴ $\frac{\pi}{2}x = \frac{(2n+1)\pi}{2}$.

x-intercepts: $((2n+1), 0)$; $n = -4, \ldots, 3$.

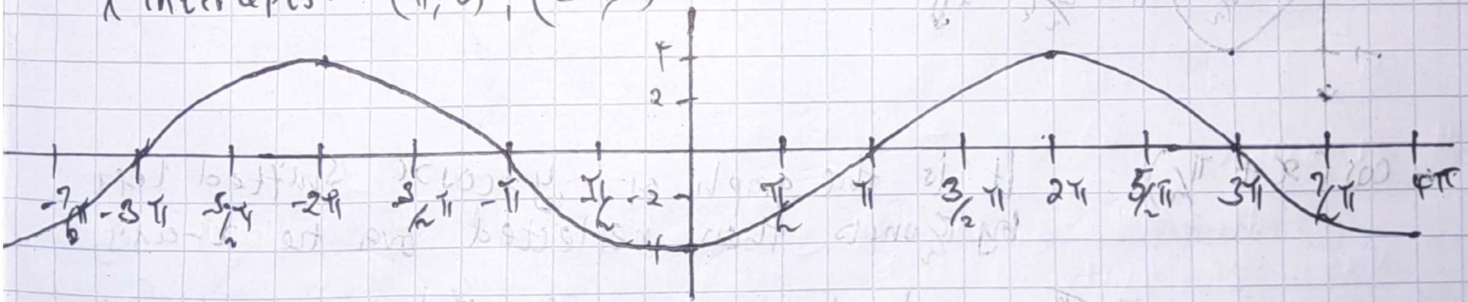
points: $(-8, 2)$, $(-4, 2)$, $(0, 2)$, $(4, 2)$, $(8, 2)$



45) $y = -4\cos(\frac{1}{2}x)$

Amplitude: $= |-4| = 4$ period $= \frac{2\pi}{1/2} = 4\pi$.

x intercepts: $(\pi, 0)$, $(3\pi, 0)$

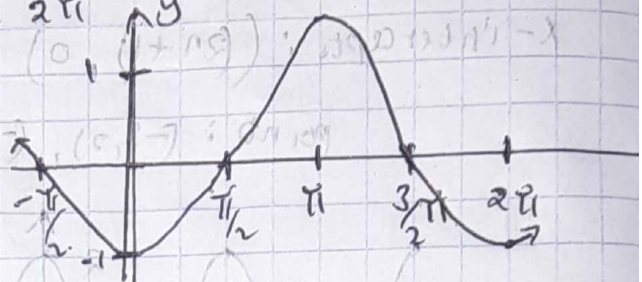


Section 4.2

1) (1-6) Match the fxn with its graphs

(1) $\sin(x - \pi/2)$

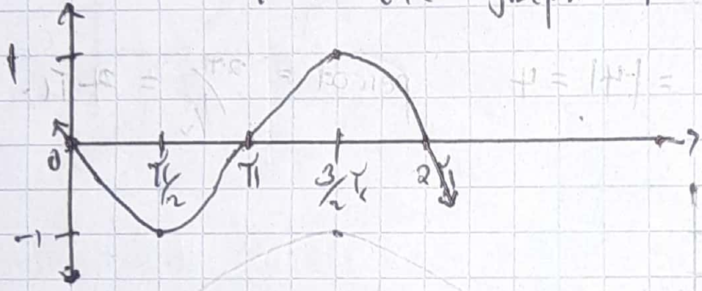
x	0	$\pi/2$	π	$\frac{3}{2}\pi$	2π
$x - \pi/2$	$-\pi/2$	0	$\pi/2$	π	$3\pi/2$
$\sin(x - \pi/2)$	-1	0	1	0	-1



$\boxed{\text{Answer: c}}$

It is the graph of $y = \sin x$ shifted to the right $\pi/2$ units

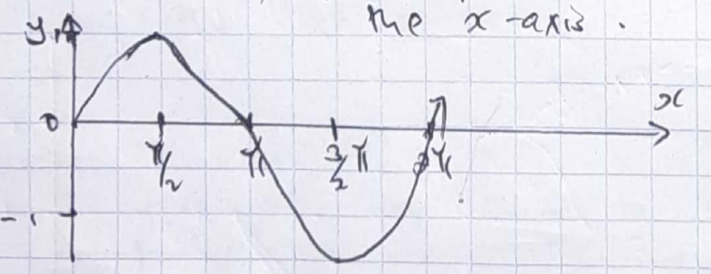
2) $\cos(x + \pi/2)$ It is the graph of $y = \cos x$ shifted left by $\pi/2$ units



Answer: b.

(3) $-\cos(x + \pi/2)$ It is the graph of $y = \cos x$ shifted left by $\pi/2$ units then reflected over the x-axis.

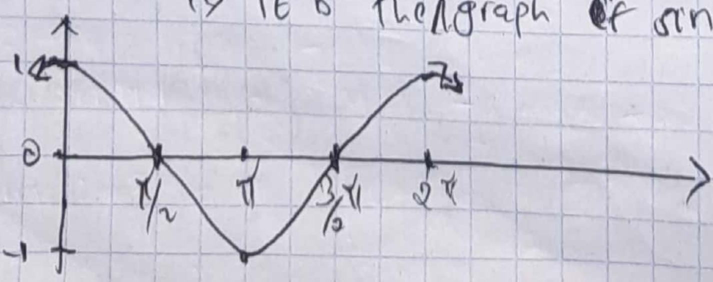
ie The graph above of $\cos(x + \pi/2)$ reflected over the x-axis.



Answer: a

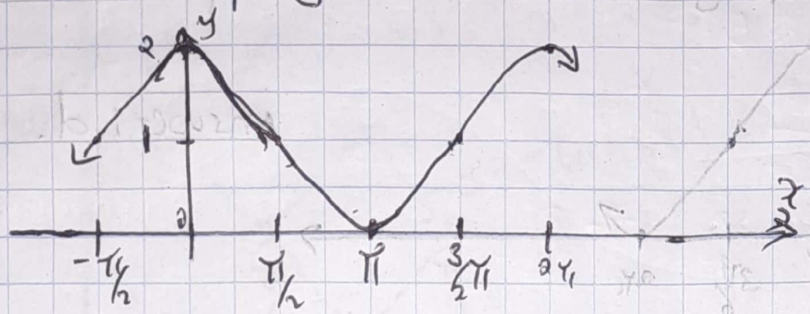
4) $-\sin(x - \pi/2)$ It is the graph of $y = \sin x$ shifted right by $\pi/2$ units and then reflected over x-axis.

ie It is the above graph of $\sin(x - \pi/2)$ reflected over x-axis



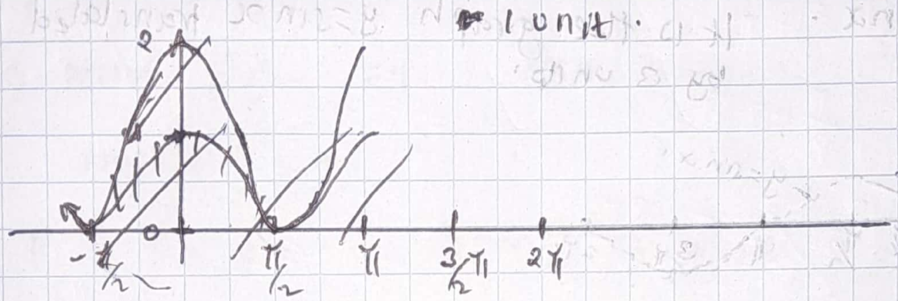
Answer: h.

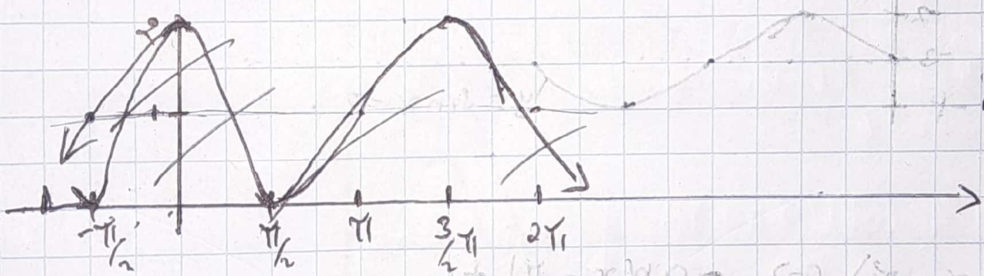
5) $\cos x + 1$ It is the graph of $y = \cos x$ translated vertically up by 1 unit.



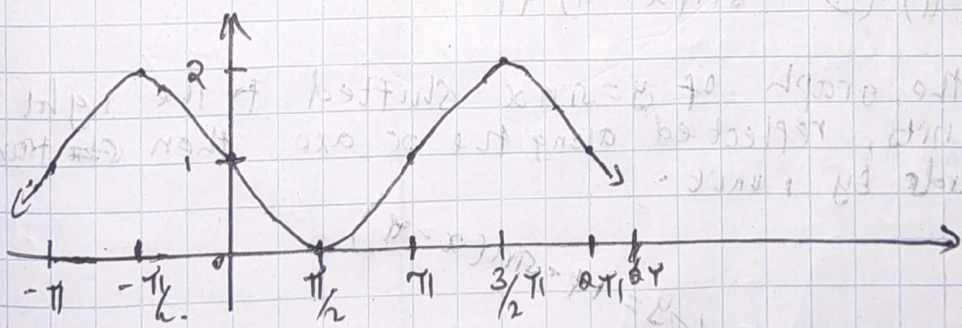
Answer: e.

$1 - \sin x = -\sin x + 1 \Leftrightarrow$ It is the graph of $y = \sin x$ reflected over the x-axis then translated upwards by 1 unit.



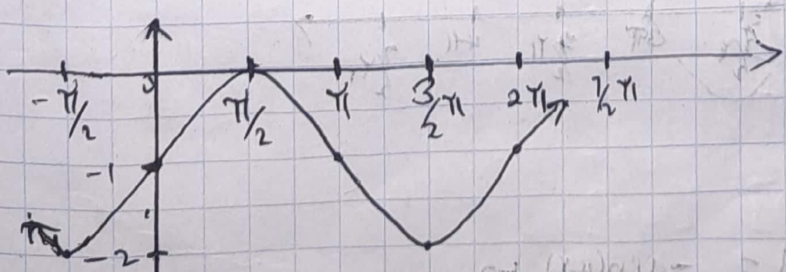


Answer: g.



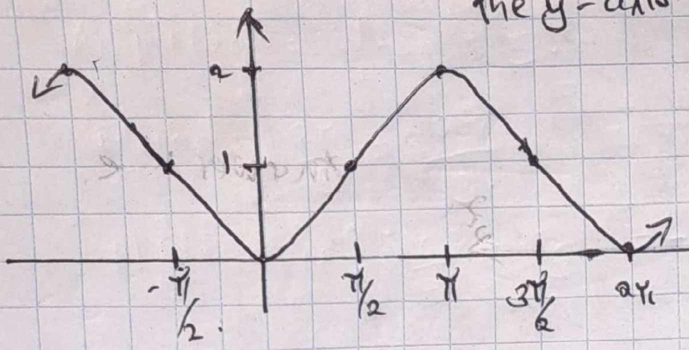
Answer: g.

7) $-1 + \sin x \Leftrightarrow \sin x - 1$. It is the graph of $y = \sin x$ translated downwards by 1 unit.



Answer: f.

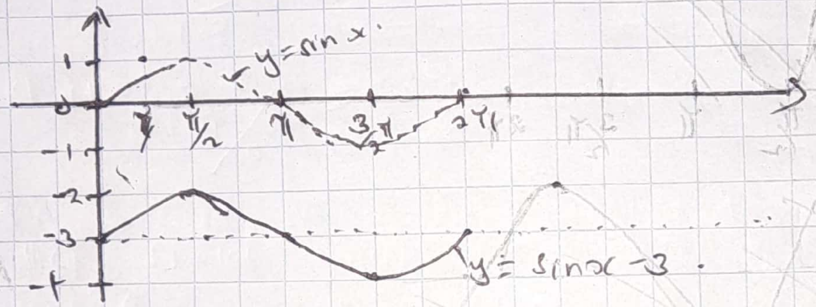
$1 - \cos x \iff -\cos x + 1$: It is the graph of $y = \cos x$ reflected along the y-axis then translated upwards by 1 unit



answer: d.

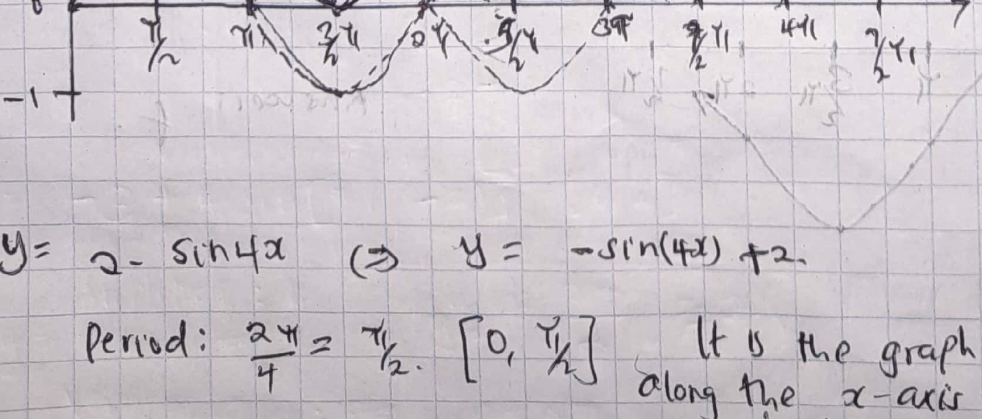
(9–24) plot the graph over 1 period.

a. $y = \cancel{3si}\ -3 + \sin x$. It is the graph $y = \sin x$ translated downwards by 3 units.



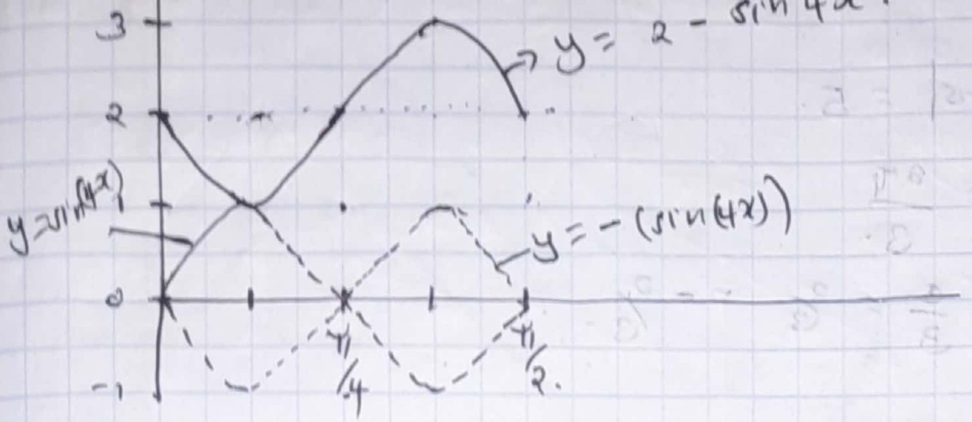
b. $y = 1 - \sin(x - \pi) \iff -\sin(x - \pi) + 1$

It is the graph of $y = \sin x$ shifted to the right by π unit, reflected along the x axis then ~~sh~~ translated upwards by 1 unit.



(7) $y = 2 - \sin 4x \iff y = -\sin(4x) + 2$.

Period: $\frac{2\pi}{4} = \frac{\pi}{2}$. $[0, \frac{\pi}{2}]$ It is the graph of $y = \sin 4x$ Reflect along the x-axis then translated upwards by 2 units.

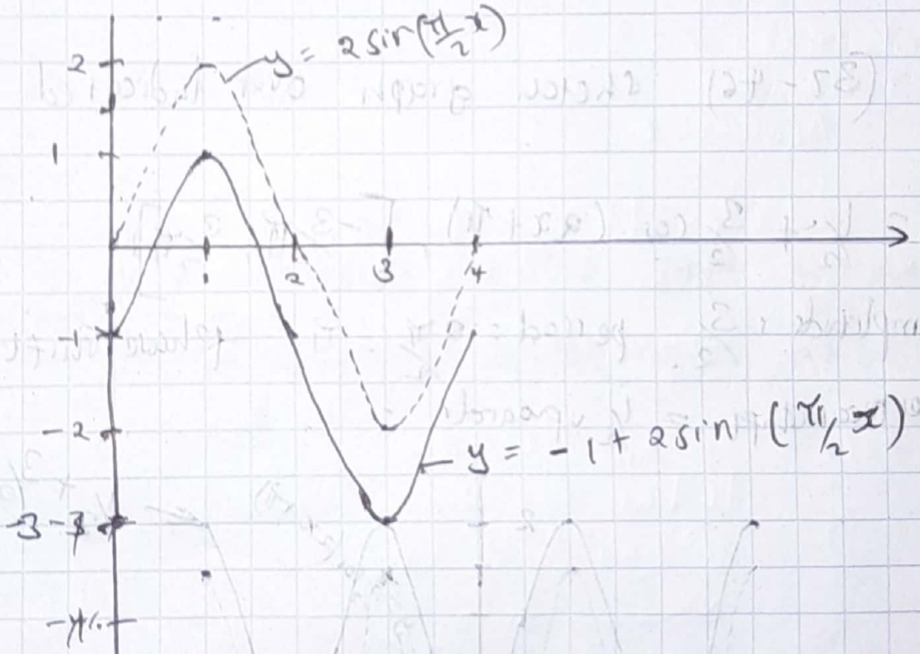


21) $y = -1 + 2\sin\left(\frac{\pi}{2}x\right)$

period $= \frac{2\pi}{\pi/2} = 4.$

Amplitude $= 2.$

It is the graph of $2\sin\left(\frac{\pi}{2}x\right)$ translated downwards by 1 unit.



(25-36) state Amplitude, period & phase shift of each sinusoidal fxn.

25) $y = -\frac{1}{3}\cos\left(x - \frac{\pi}{2}\right)$

$y = A\cos(Bx + c)$

Amplitude $= \left|-\frac{1}{3}\right| = \frac{1}{3}.$

period $= \frac{2\pi}{|B|} = \frac{2\pi}{1} = 2\pi.$

Phase shift $= \cancel{|c|} = \cancel{\left|-\frac{\pi}{2}\right|} = \frac{\pi}{2} = c = \frac{\pi}{2}$ to the left.

29. $y = -5\cos(3x+2)$

Amplitude $= |-5| = 5.$

period $= \frac{2\pi}{B} = \frac{2\pi}{3}.$

Phase shift $= \frac{c}{B} = \frac{2}{3} = -\frac{2}{3}.$

33 $y = -3\sin\left(2x - \frac{2\pi}{4}\right)$

Amplitude $= |-3| = 3$

period $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi.$

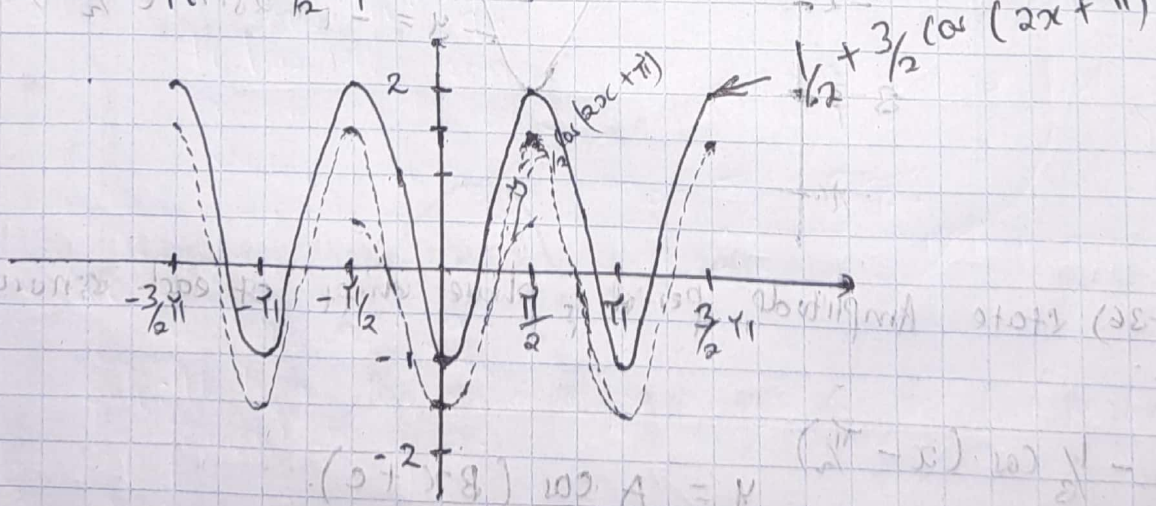
Phase shift $= 3\left(\frac{\frac{2}{4}\pi}{2}\right) = \frac{3\pi}{8}$

(37-46) sketch graph over Indicated Interval.

37) $y = \frac{1}{2} + \frac{3}{2}\cos(2x+\pi), \left[-\frac{3}{2}\pi, \frac{3}{2}\pi\right]$.

Amplitude : $\frac{3}{2}$. period $= \frac{2\pi}{2} = \pi$. phase shift $= -\frac{\pi}{2}$.

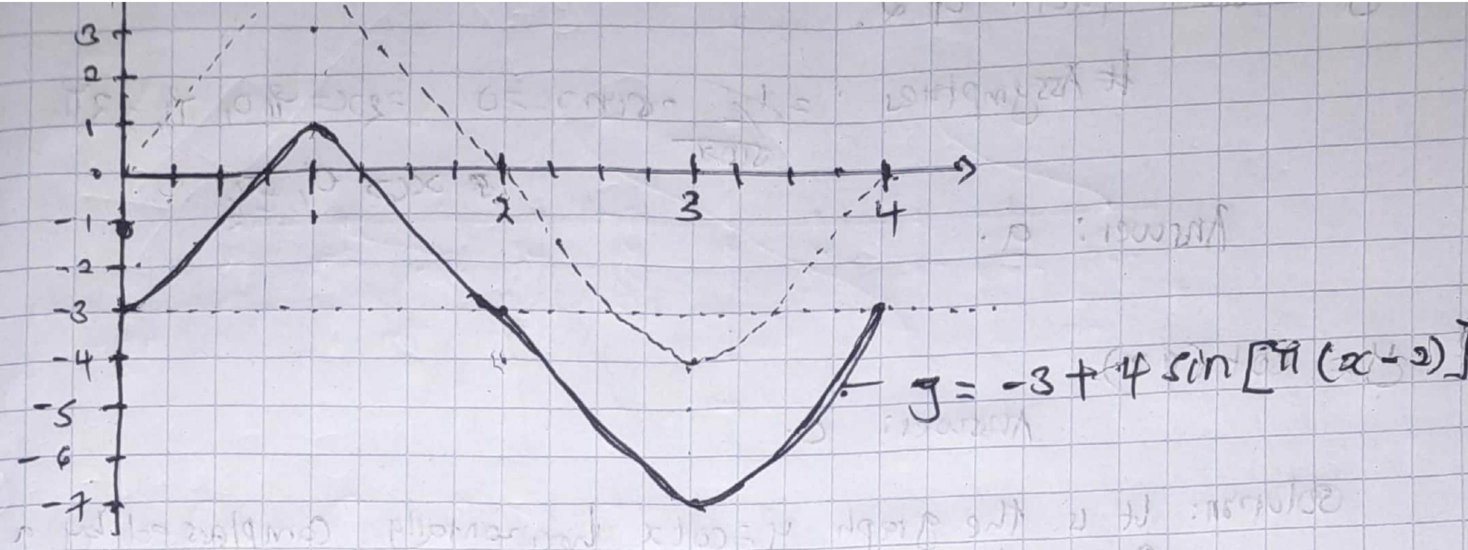
vertical shift $= \frac{1}{2}$ upwards.



41) $y = -3 + 4\sin[\pi(x-2)], [0, 4]$

Amplitude : 4 period: $\frac{2\pi}{\pi} = 2$. phase shift $= 2$

Vertical shift : 3 units Downwards.



Section 4.3

(1 - 8) Match each fxn to the graph. (a - h)

1) $y = -\tan x$.

Answer: b

Solution: It is a reflection of the graph $y = \tan x$. over the x-axis.

It has an ~~assym~~ vertical assymtotes at $x = \pm \frac{\pi}{2}$

assymptote: $= -\frac{\sin\theta}{\cos\theta}$

~~cos~~ $\cos\theta = 0$ for $\pm \frac{\pi}{2}$.

2) $y = -\csc x$.

Answer: f.

Solution: It is the reflection of the graph $y = \csc x$ over the x ~~a~~ -axis and has vertical assymptote at $x = \pm \pi$

Assymptote : $-\overset{\csc}{\cancel{\cos}}\, x = -\frac{1}{\cancel{\cos}\ \sin x}$. where $\sin x \neq 0$.

$\sin x = 0$ for $\pm \pi$ hence it is te assymptote

3) $y = \sec(2x)$

~~# $y = \sec 2x$~~ Answer: h.

It is the graph of $\sec x$ compressed horizontally by a scale factor of 2.

Vertical assymptotes: $\frac{1}{\cos(2x)} \neq 0$ $\cos \cancel{2x} = 0$ $\frac{\pi}{2}, \frac{3\pi}{2}$ $= \frac{\pi}{2} \div 2, \frac{3\pi}{2} \div 2 \Longleftrightarrow \frac{\pi}{4}, \frac{3\pi}{4}$

4) $y = \csc(2x)$ It is the graph of $y = \csc x$ horizontally compressed by a scale factor of 2. (soln)

~~#~~ Assymptotes : $= \frac{1}{\sin 2x}$; $\sin x = 0$ $= 2x = \cancel{4\pi}\ 0, \pi, 2\pi$

$\cancel{\#}\ x \Leftrightarrow 0, \pm \frac{\pi}{2}$

Answer: a.

5) $y = \cot(\pi x)$

Answer: c

Solution: It is the graph $y = \cot x$ horizontally complarsed by a factor π.

vertical assympoter : $\tan \frac{\pi x}{\cancel{2\pi}} = 0$. $\frac{\pi x}{\pi} = 0, \pm \pi$

$\cancel{x} = 0, \pm 1$

6) $y = -\cot(\pi x)$

Answer: e

Solution: It is the graph of $y = \cot x$ reflected over the x-axis.

Assymptotes: $\cancel{x} = 0, \pm 1$

7) $y = 3\sec x$. Answer: d.

It has the same shape as $y = \sec x$ but has an amplitude of 3.

~~verti~~ vertical Assymptoter: $x = \frac{\pi}{2}, \frac{3}{2}\pi$.

8) $y = 3\csc x$

Answer: g.

Soln: It has the same shape as $y = \csc x$ but it has an amplitude of 3.

Vertical assymptotes: $0, \pi, 2\pi$.

(9 – 18) : Period & phase shift

9) $y = \tan(x + \pi)$ $y = a\tan(bx \pm c)$

Period $= \frac{\cancel{2\pi}}{1}$ ~~$= 2\pi$~~ $\frac{\pi}{|b|} = \frac{\pi}{1} = \pi$

phase shift $= -CB = -(-\pi \times 1) = \pi.$

11) $y = \sec(2x - \pi/2)$

period $= \frac{2\pi}{|B|} = \frac{2\pi}{2} = \pi.$

phase shift $= \frac{\pi/2}{2} = \pi/4.$

13) $y = \csc[3(x - \pi/6)] = \csc(3x - \pi/2)$

period $= \frac{2\pi}{B}\ \frac{2\pi}{3}$

phase shift $= \frac{\pi/2}{3} = \pi/6.$

15) ~~Period~~ $y = \tan(0.5x + 2\pi)$

period $= \frac{\pi}{B} = \frac{\pi}{1/2} = 2\pi$

phase shift $= -\frac{2\pi}{0.5} = \frac{-2\pi}{1/2} = -4\pi.$

17) $y = -3\cot(\pi x - \pi/2)$

period $= \frac{\pi}{B} = \frac{\pi}{\pi} = 1$

phase shift $= -\left(\frac{-\pi/2}{\pi}\right) = 1/2.$

(19-30) Graph the fns over the Indicated Intervals:

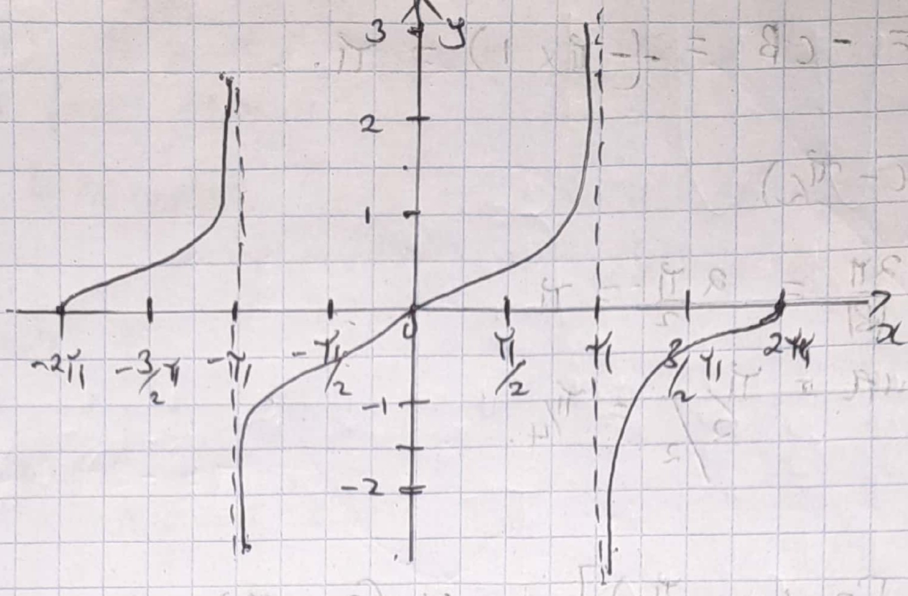
19. $y = \tan(\frac{1}{2}x) \quad -2\pi \le x \le 2\pi$

period $= \frac{\pi}{1/2} = 2\pi$ No reflection, No phase.

$\tan x = \frac{\sin x}{\cos x}$.

x-intercepts: $\frac{1}{2}x = 0, \pm\pi \Rightarrow x = 0, \pm 2\pi$. : $(0,0)$ $(2\pi, 0)$, $(-2\pi, 0)$

vertical assymptote: $\cos(\frac{1}{2}x) = 0$, $\frac{1}{2}x = \pm\frac{\pi}{2}$, $x = \pm\pi$



21) $y = -\cot(2\pi x)$, $-1 \le x \le 1$

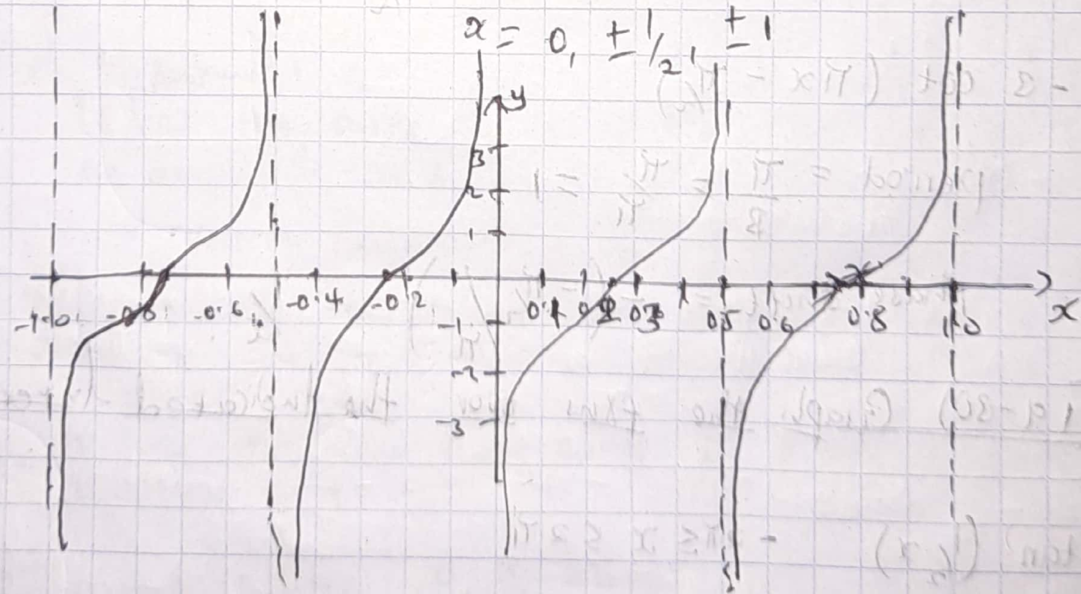
period: $\frac{\pi}{2\pi} = \frac{1}{2}$. Reflection about x-axis

x intercepts: $\cos(2\pi x) = 0$, $2\pi x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$.

$x = \pm\frac{1}{4}, \pm\frac{3}{4}$.

vertical asymptote:
$\sin(2\pi x) = 0$, $(2\pi x) = 0, \pm\pi, \pm 2\pi$

$x = 0, \pm\frac{1}{2}, \pm 1$



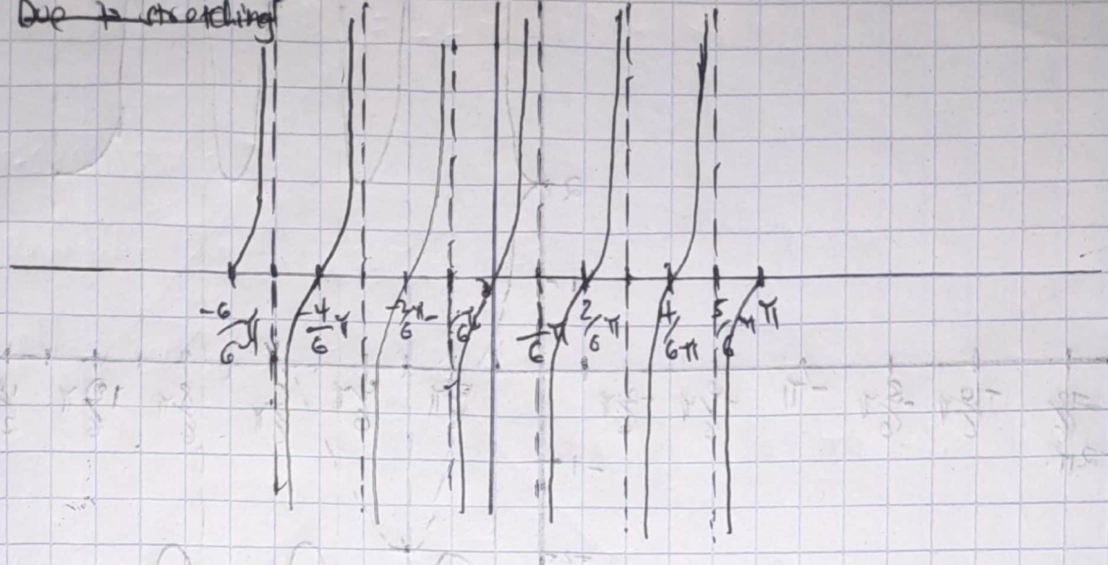
23) ~~y=2 tan~~ $y = 2\tan 3x$. $(-\pi \le x \le \pi)$

period = $\frac{\pi}{3}$

x Intercepts: $3x = 0, \pm\pi, \pm 2\pi, \pm 3\pi$, $x = 0, \pm\frac{\pi}{3}, \frac{2}{3}\pi, \pm\pi$.

Asymptotes: $3x = \pm\frac{\pi}{2}, \pm\frac{3}{2}\pi, \pm\frac{5}{2}\pi$, $x = \pm\frac{\pi}{6}, \pm\frac{3}{6}\pi, \pm\frac{5}{6}\pi$

~~Due to stretching~~



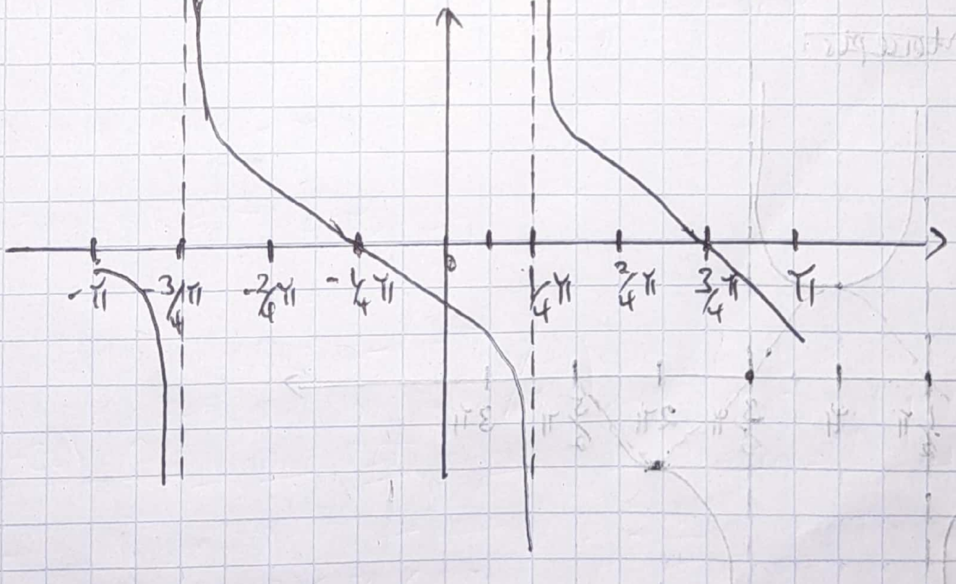
27) $y = \cot\left(x - \frac{\pi}{4}\right)$ $[-\pi, \pi]$

Period $= \frac{\pi}{1} = \pi$.   phase shift $=$ ~~Right~~ $-\left(\frac{-\pi/4}{1}\right) = \frac{\pi}{4}$ (Right)

x intercept $\cos\left(x - \frac{\pi}{4}\right) = 0$, $x - \frac{\pi}{4} = \pm\frac{\pi}{2}$, $x = \frac{3}{4}\pi$, $-\frac{\pi}{4}$.

vertical Asymptote.

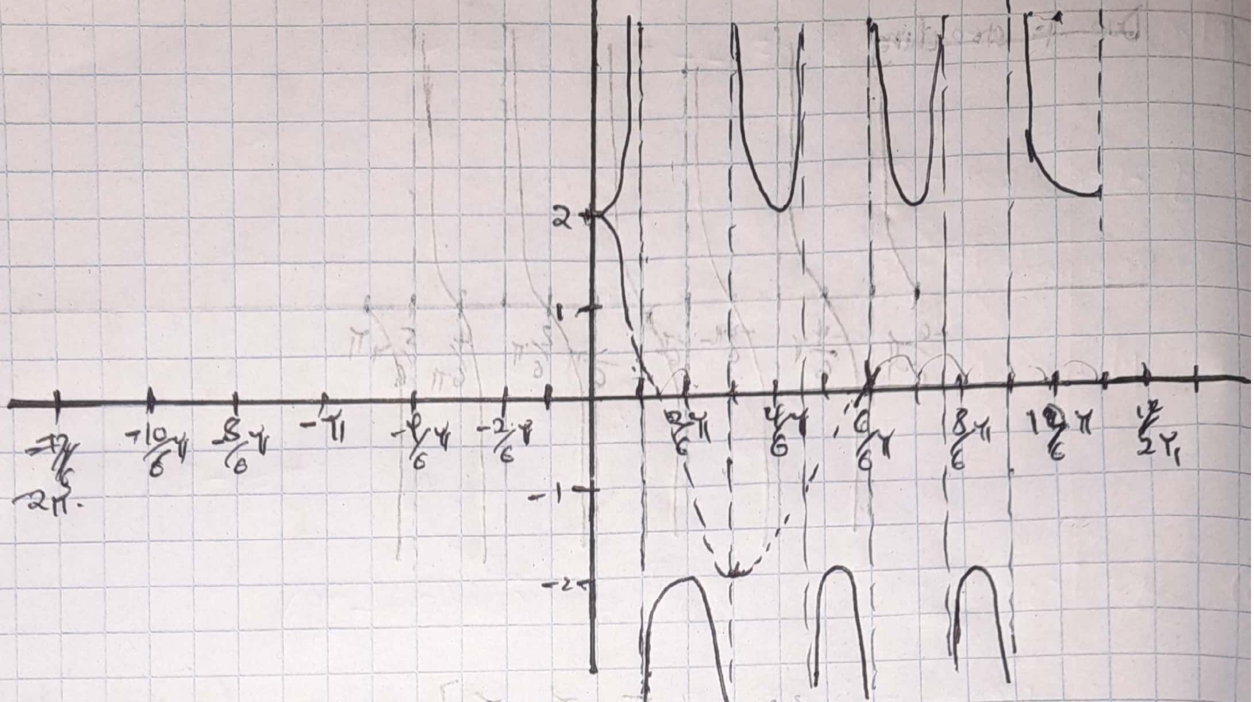
$\sin\left(x - \frac{\pi}{4}\right) = 0$, $x - \frac{\pi}{4} = 0, \pm\pi$, $x = \frac{\pi}{4}, -\frac{3}{4}\pi, \frac{5}{4}\pi$.



35) $y = 2\sec(3x)$, $0 \le x \le 2\pi$

x intercept : $3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5}{2}\pi, \frac{7}{2}\pi, \frac{9}{2}\pi, \frac{11}{2}\pi$.

$x = \frac{\pi}{6}, \frac{3}{6}\pi, \frac{5}{6}\pi, \frac{7}{6}\pi, \frac{9}{6}\pi, \frac{11}{6}\pi$.

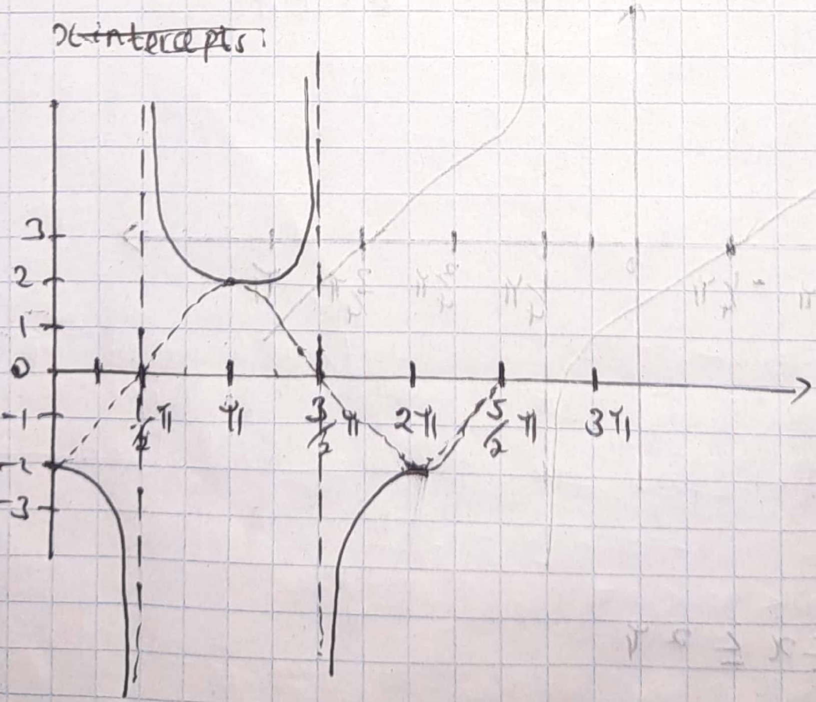


43 $y = 3 - 2\sec\left(x - \frac{\pi}{2}\right)$

Step 1: $y = -2\sec\left(x - \frac{\pi}{2}\right)$

x intercepts: $x - \frac{\pi}{2} = \frac{\pi}{2}, \frac{3}{2}\pi$, $x = \pi, 2\pi$

~~x intercepts:~~



Step 2 : Translate.